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Radiative rare kaon decays[†]

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Abstract

We discuss theoretical issues in radiative rare kaon decays. The interest is twofold: to extract useful short-distance information and understand the underlying dynamics. We emphasize channels where either we can understand non-perturbative aspects of QCD or there is a chance to test the Standard Model.

1 Introduction

Kaon decays are an important place to study non-perturbative aspects of QCD and test the Standard Model. Indeed some channels are completely dominated by long-distance dynamics, such as the CP-conserving amplitude for $K \rightarrow \pi\pi$ and others, like $K \rightarrow \pi\nu\bar{\nu}$ [1, 2], which are described in terms of pure short-distance physics. In this review we will be mostly concerned with kaon decays involving electromagnetic interactions and thus long-distance phenomena are not negligible. However, as we shall see, it is still possible in these channels to extract the short-distance component with an accurate analysis. Indeed there are plenty of motivations to look for new physics (NP) in these kaon decays [3]. The channels which will be considered here are $K_S \rightarrow \gamma\gamma$, $K \rightarrow \pi\gamma\gamma$, $K \rightarrow \pi\ell^+\ell^-$, $K \rightarrow \pi\pi\gamma$ and $K_L \rightarrow \mu\bar{\mu}$. Experiments at CERN, Fermilab, Brookhaven, KLOE [4]-[9] are and will be also providing a large amount of data to further motivate this research. QCD at low energy will be studied in the framework of chiral perturbation theory (ChPT) [10]-[12]. The $\Delta S = 1$ weak Lagrangian is expanded in powers of external momenta and masses: there is only one $\mathcal{O}(p^2)$ operator for the $\Delta I = 1/2$ and $\Delta I = 3/2$, with coefficients

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G_8 and G_{27} respectively, determined from $K \rightarrow \pi\pi$ transitions. The $\mathcal{O}(p^4)$ Lagrangian has many operators W_i , and corresponding coefficients N_i [13]:

$$\mathcal{L}_{\Delta S=1}^{(p^4)} = G_8 F^2 \sum_{i=1}^{37} N_i W_i \quad (1)$$

and although there are already interesting tests at this level, as we shall see, it is clear that further assumptions are needed in order to be reasonably predictive, typically of vector meson dominance and $1/N$ [13]-[15].

2 $K_S \rightarrow \gamma\gamma$

$K_S \rightarrow \gamma\gamma$ has vanishing short-distance contributions [16], so it is a pure long-distance phenomenon; since the external particles are neutral there is no $\mathcal{O}(p^2)$ amplitude. For the same reason, if we write down the $\mathcal{O}(p^4)$ counterterm structure, $F_{\mu\nu} F^{\mu\nu} \langle \lambda_6 Q U^+ Q U \rangle$, this gives a vanishing contribution (we use the standard chiral notation as in Ref. [11]); this implies that at $\mathcal{O}(p^4)$: i) we have only

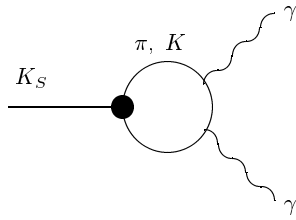


Figure 1: $K_S \rightarrow \gamma\gamma$ [17]

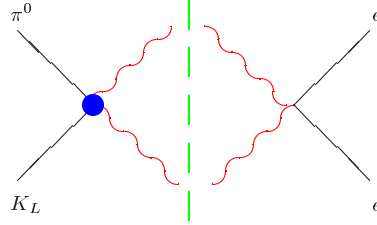


Figure 2: CP-conserving contribution to $K_L \rightarrow \pi^0 e^+ e^-$

a loop contribution in Fig. 1 and ii) this contribution is scale-independent; it is completely predicted by the $K_S \rightarrow \pi\pi$ amplitude [17] and can be compared with the recent NA48 result [18]:

$$B(K_S \rightarrow \gamma\gamma)_{\text{ChPT}} = 2.1 \times 10^{-6} \quad B(K_S \rightarrow \gamma\gamma)_{\text{NA48}} = (2.6 \pm 0.4) \times 10^{-6}. \quad (2)$$

$\mathcal{O}(p^6)_{\text{CT}}$, as the structure

$$F^{\mu\nu} F_{\mu\nu} \langle \lambda_6 Q^2 \mu M U^+ \rangle \quad , \quad (3)$$

in principle can modify Eq.(2), but chiral power counting suggests $A^{(6)}/A^{(4)} \sim m_K^2/(4\pi F_\pi)^2 \sim 0.2$. In fact the potentially large Vector Meson (VMD) contributions, which could alter this relation as in $K_L \rightarrow \pi^0 \gamma\gamma$, are absent in this channel. Since higher order π -loop corrections are small [19], we can look at the theoretical prediction in Eq.(2) as a test of non-VMD contributions.

3 $K \rightarrow \pi\gamma\gamma$ decays and the CP-conserving $K_L \rightarrow \pi^0\ell^+\ell^-$

$K_L \rightarrow \pi^0\ell^+\ell^-$ is a classical example of how our control on low energy theory may help to disentangle short-distance physics. In fact the effective current \otimes current structure of weak interactions obliges short-distance contributions to $K_L \rightarrow \pi^0\ell^+\ell^-$, analogously to $K_L \rightarrow \pi^0\nu\bar{\nu}$, to be direct CP-violating [1, 20]. However, differently from the neutrino case, $K_L \rightarrow \pi^0\ell^+\ell^-$ receives also non-negligible long-distance contributions: i) indirect CP-violating from one-photon exchange, discussed in the next section, and ii) CP-conserving from two-photon exchange in Fig. 2, where the photons can be on-shell (two-photon discontinuity) and thus directly related to the observable $K_L \rightarrow \pi^0\gamma\gamma$ decay, or off-shell and then a form factor should be used [21]. We will comment in the conclusions on possible ways to avoid the potential large background contribution from $K_L \rightarrow e^+e^-\gamma\gamma$ [22]. The present bounds from KTeV [6, 23, 24] are

$$B(K_L \rightarrow \pi^0 e^+ e^-) < 5.1 \times 10^{-10} \quad \text{and} \quad B(K_L \rightarrow \pi^0 \mu^+ \mu^-) < 3.8 \times 10^{-10}. \quad (4)$$

The general amplitude for $K_L(p) \rightarrow \pi^0\gamma(q_1)\gamma(q_2)$ can be written in terms of two Lorentz and gauge invariant amplitudes $A(z, y)$ and $B(z, y)$:

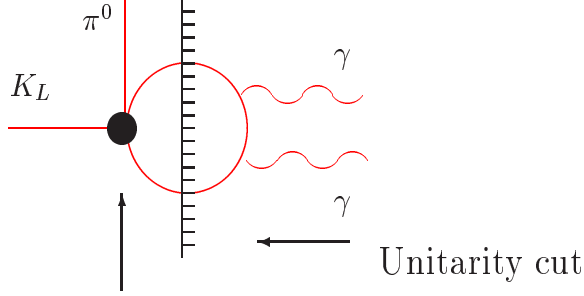
$$\begin{aligned} \mathcal{A}(K_L \rightarrow \pi^0\gamma\gamma) = & \frac{G_8\alpha}{4\pi}\epsilon_{1\mu}\epsilon_{2\nu}\left[A(z, y)(q_2^\mu q_1^\nu - q_1 \cdot q_2 g^{\mu\nu}) + \right. \\ & \left. + \frac{2B(z, y)}{m_K^2}(p \cdot q_1 q_2^\mu p^\nu + p \cdot q_2 p^\mu q_1^\nu - p \cdot q_1 p \cdot q_2 g^{\mu\nu} - q_1 \cdot q_2 p^\mu p^\nu)\right], \end{aligned} \quad (5)$$

where $y = p(q_1 - q_2)/m_K^2$ and $z = (q_1 + q_2)^2/m_K^2$. Then the double differential rate is given by

$$\frac{\partial^2\Gamma}{\partial y \partial z} \sim [z^2 |A + B|^2 + \left(y^2 - \frac{\lambda(1, r_\pi^2, z)}{4}\right)^2 |B|^2], \quad (6)$$

where $\lambda(a, b, c)$ is the usual kinematical function and $r_\pi = m_\pi/m_K$. Thus in the region of small z (collinear photons) the B amplitude is dominant and can be determined separately from the A amplitude. This feature is crucial in order to disentangle the CP-conserving contribution $K_L \rightarrow \pi^0 e^+ e^-$. In fact the lepton pair in Fig. 2 produced by photons in S -wave, like an $A(z)$ -amplitude, are suppressed by the lepton mass while the photons in $B(z, y)$ are also in D -wave and so the resulting $K_L \rightarrow \pi^0 e^+ e^-$ amplitude, $A(K_L \rightarrow \pi^0 e^+ e^-)_{CPC}$, does not suffer from the electron mass suppression [25, 26].

The leading $\mathcal{O}(p^4)$ $K_L \rightarrow \pi^0\gamma\gamma$ amplitude [27] is affected by two large $\mathcal{O}(p^6)$ contributions: i) the full unitarity corrections from $K \rightarrow 3\pi$ [29, 30] in Fig. 3



$$A(K \rightarrow 3\pi) = a + b Y + c Y^2 + d X^2$$

Figure 3: $K_L \rightarrow \pi^0 \gamma \gamma$: unitarity contributions from $K \rightarrow 3\pi$: X, Y Dalitz variables [29, 30]

and ii) local contributions. Fig. 3 enhances the $\mathcal{O}(p^4)$ branching ratio by 40% and generates a B -type amplitude. At this order there are three independent counterterms, as the one in Eq. (3), with the unknown coefficients α_1, α_2 and β leading to contributions to A and B in Eq. (5) [30]:

$$A_{\text{CT}} = \alpha_1(z - r_\pi^2) + \alpha_2, \quad B_{\text{CT}} = \beta. \quad (7)$$

If we assume VMD [28, 31], these couplings are related in terms of one constant, a_V :

$$\alpha_1 = \frac{\beta}{2} = -\frac{\alpha_2}{3} = -4a_V. \quad (8)$$

Though chiral counting suggests $\alpha_i, \beta \sim 0.2$, VMD enhances this typical size. Actually a model, FMV, describing weak interactions of pseudoscalars (ϕ 's) with vectors, $\mathcal{L}_W^{FMV}(\phi, V^\mu)$, based on factorization and couplings fixed by the Wilson coefficient of the Q_- operator, predicts ¹ the size and the sign of the weak VMD couplings [32]:

$$\mathcal{L}_W^{FMV}(\phi, V^\mu) \implies a_V = -0.6. \quad (9)$$

As we can see from Fig. 3 the spectrum at low z is very sensitive to the value of a_V , or more generally to the size of the amplitude B in Eq. (5). Recently Gabbiani and Valencia [33] suggested to fit the experimental z -spectrum (and the rate) with all three parameters in Eq. (7): their preferred solution for B and consequently $B(K_L \rightarrow \pi^0 e^+ e^-)_{CPC}$ is very large in fact at low diphoton

¹Due to the better ultraviolet behaviour, as discussed in Section 6, we consider only FMV contributions for the predictions.

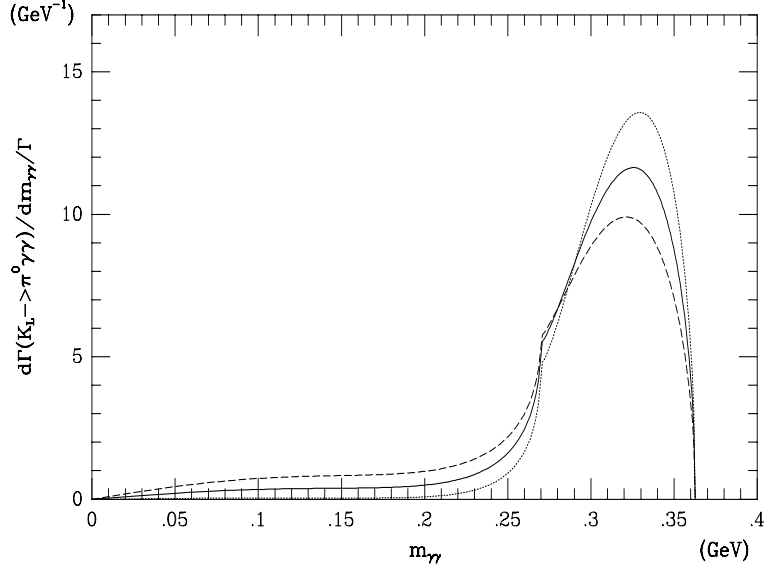


Figure 4: $K_L \rightarrow \pi^0 \gamma \gamma$ diphoton-invariant mass spectrum for different values of a_V : -0.4 (dotted curve), -0.7 (full curve), -1 (dashed curve) [32].

invariant mass it is consistent with our plot in Fig. 3 with $a_V : -1$. The recent data from NA48 [4] measure this region extremely well and exclude this possibility by finding $a_V : -0.46 \pm 0.05$.

$$\text{NA48} \Rightarrow B(K_L \rightarrow \pi^0 e^+ e^-)_{CPC}^{\gamma\gamma \text{ on-shell}} \sim 1 \times 10^{-12}. \quad (10)$$

As a result we think that the size $B(K_L \rightarrow \pi^0 e^+ e^-)_{CPC}^{\gamma\gamma \text{ on-shell}}$, is an issue that can be established firmly from the $K_L \rightarrow \pi^0 \gamma \gamma$ spectrum. More disturbing is the dependence on the form factor when the two intermediate photons are off-shell. More theoretical work is needed and probably a partial answer can come from the measurement of $K_L \rightarrow \pi^0 \gamma \gamma^*$ [34, 35].

The $K^+ \rightarrow \pi^+ \gamma \gamma$ channel can be studied in much the same way as $K_L \rightarrow \pi^0 \gamma \gamma$ and might be interesting for future E949 [6] and NA48b [9] experiments. Since the K^+ is not CP-eigenstate, in addition to A and B in eq.(5), also a helicity amplitude with $CP=-1$ and photons in the P -wave is allowed but found small [26]. The A and B amplitudes receive: i) a $\pi\pi$ -loop contribution analogous to Fig. 3 [26, 36] and ii) $\mathcal{O}(p^4)$ local contributions from Eq. (1), \hat{c} [26], and small $\mathcal{O}(p^6)$ VMD contributions [36]. BNL787 with 31 events has measured $B(K^+ \rightarrow \pi^+ \gamma \gamma) = (6 \pm 1.6) \times 10^{-7}$ and $\hat{c} = (1.8 \pm 0.6)$ [37], which has interesting dynamical implications [13, 14].

4 $K^\pm \rightarrow \pi^\pm \ell^+ \ell^-$ and $K_S \rightarrow \pi^0 \ell^+ \ell^-$

The CP-conserving decays $K^\pm(K_S) \rightarrow \pi^\pm(\pi^0)\ell^+\ell^-$ are dominated by the long-distance process $K \rightarrow \pi\gamma \rightarrow \pi\ell^+\ell^-$ [38]. The decay amplitudes can in general be written in terms of one form factor $W_i(z)$ ($i = \pm, S$):

$$A(K_i \rightarrow \pi^i \ell^+ \ell^-) = -\frac{e^2}{M_K^2 (4\pi)^2} W_i(z) (k+p)^\mu \bar{u}_\ell(p_-) \gamma_\mu v_\ell(p_+) , \quad (11)$$

$z = q^2/M_K^2$; $W_i(z)$ can be decomposed as the sum of a polynomial piece plus a non-analytic term, $W_i^{\pi\pi}(z)$, generated by the $\pi\pi$ loop, analogously to the one in Fig. 3 for $K_L \rightarrow \pi^0 \gamma\gamma$, completely determined in terms of the physical $K \rightarrow 3\pi$ amplitude [39]. Keeping the polynomial terms up to $\mathcal{O}(p^6)$ we can write

$$W_i(z) = G_F M_K^2 (a_i + b_i z) + W_i^{\pi\pi}(z) , \quad (12)$$

where the parameters a_i and b_i parametrize local contributions starting respectively at $\mathcal{O}(p^4)$ and $\mathcal{O}(p^6)$. Recent data on $K^+ \rightarrow \pi^+ e^+ e^-$ and $K^+ \rightarrow \pi^+ \mu^+ \mu^-$ by BNL-E865 [40] have been successfully² fitted using Eq. (12) and lead to

$$a_+ = -0.587 \pm 0.010, \quad b_+ = -0.655 \pm 0.044 . \quad (13)$$

Recently HyperCP [7] has measured the CP-violating width charge asymmetry in $K^\pm \rightarrow \pi^\pm \mu^+ \mu^-$ and it has found that it is consistent with 0 at 10% level. Though the CKM prediction with accurate cuts is $\sim 10^{-4}$ [39], we are beginning to test new physics affecting the operator $\bar{s}d\bar{\mu}\mu$. The experimental size of the ratio b_+/a_+ exceeds the naive dimensional analysis estimate $b_+/a_+ \sim \mathcal{O}[M_K^2/(4\pi F_\pi)^2] \sim 0.2$, but can be explained by a large VMD contribution. Chiral symmetry alone does not allow us to determine the unknown couplings a_S and b_S in terms of a_+ and b_+ [38, 39]. Neglecting the $\Delta I = 3/2$ suppressed non-analytic term $W_S^{\pi\pi}(z)$, we obtain [39]

$$B(K_S \rightarrow \pi^0 e^+ e^-) = [46.5a_S^2 + 12.9a_S b_S + 1.44b_S^2] \times 10^{-10} \approx 5 \times 10^{-9} \times a_S^2 , \quad (14)$$

The recent experimental information $B(K_S \rightarrow \pi^0 e^+ e^-) < 1.4 \times 10^{-7}$ [42] let us derive the bound $|a_S| \leq 5.3$; NA48 [9] and maybe KLOE [8] will assess in the near future the value of this branching at the least for values of a_S of order 1. Of course even a strong bound is relevant, since it will ensure that this contribution is not dangerous to measure direct CP violation in $K_L \rightarrow \pi^0 e^+ e^-$. We remark that even a sizeable a_S : $a_S < -0.5$ or $a_S > 1$, will lead to an

²For an alternative description of the data see [41].

interesting interference:

$$B(K_L \rightarrow \pi^0 e^+ e^-)_{CPV} = \left[15.3 a_S^2 - 6.8 \frac{\Im \lambda_t}{10^{-4}} a_S + 2.8 \left(\frac{\Im \lambda_t}{10^{-4}} \right)^2 \right] \times 10^{-12}, \quad (15)$$

where $\lambda_t = V_{td}V_{ts}$. The sign of the interference term is model-dependent, but,

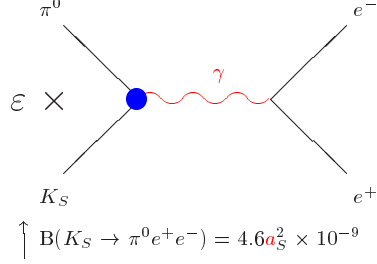


Figure 5: Indirect CP violation contribution to $B(K_L \rightarrow \pi^0 e^+ e^-)$

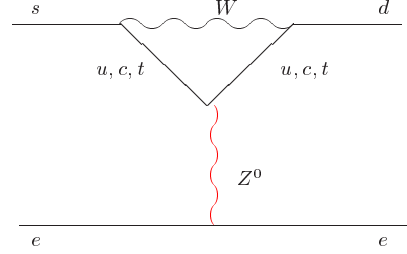


Figure 6: Typical direct CP violation contribution to $B(K_L \rightarrow \pi^0 e^+ e^-)$

however is not a problem to determine $\Im \lambda_t$ accurately (up to a discrete ambiguity).

5 $K \rightarrow \pi\pi\gamma$

We can decompose $K(p) \rightarrow \pi(p_1)\pi(p_2)\gamma(q)$ amplitudes, according to gauge and Lorentz invariance, in electric (E) and magnetic (M) terms [11, 43, 45]

$$A(K \rightarrow \pi\pi\gamma) = \varepsilon_\mu(q) [E(z_i)(p_1 \cdot q p_2^\mu - p_2 \cdot q p_1^\mu) + M(z_i)\epsilon^{\mu\nu\rho\sigma} p_{1\nu} p_{2\rho} q_\sigma] / m_K^3, \quad (16)$$

where $z_i = p_i \cdot q / m_K^2$ and $z_3 = p_K \cdot q / m_K^2$. In the electric transitions one generally separates the bremsstrahlung amplitude E_B , theoretically predicted firmly by the Low theorem in terms of the non-radiative amplitude and enhanced by the $1/E_\gamma$ behaviour. Summing over photon helicities, there is no interference among electric and magnetic terms: $d^2\Gamma/(dz_1 dz_2) \sim |E(z_i)|^2 + |M(z_i)|^2$. At the lowest order, (p^2), one obtains only E_B . Magnetic and electric direct emission amplitudes, appearing at $\mathcal{O}(p^4)$, can be decomposed in a multipole expansion (see Refs.[11, 43, 44, 45]). In the table below we show the present experimental status with the reason for the suppression of the bremsstrahlung amplitude and the leading multipoles.

Decay	$BR(\text{bremsstrahlung})$	$BR(\text{direct emission})$
$K_S \rightarrow \pi^+\pi^-\gamma$ $E_\gamma^* > 50\text{MeV}$	$(1.78 \pm 0.05) \times 10^{-3}$	$< 6 \times 10^{-5}(E1)$
$K_L \rightarrow \pi^+\pi^-\gamma$ $E_\gamma^* > 20\text{MeV}$	$(1.49 \pm 0.08) \times 10^{-5}$ (CP violation)	$(3.09 \pm 0.06) \times 10^{-5}$ $M1, E2$
$K^\pm \rightarrow \pi^\pm\pi^0\gamma$ $T_{\pi^+}^* = (55 - 90)\text{MeV}$	$(2.57 \pm 0.16) \times 10^{-4}$ ($\Delta I = 3/2$)	$(4.72 \pm 0.77) \times 10^{-6}$ $E1, M1$

$K_S \rightarrow \pi^+\pi^-\gamma$. This channel might be interesting for KLOE [8] and NA48 [9]: only for large photon energy might the dynamical interesting $E1$ – E_B interference be observed over the pure bremsstrahlung rate [47]. The relevant $\mathcal{O}(p^4)$ counterterm combination to $E1$ in this channel is related by chiral symmetry to the one contributing to $K^+ \rightarrow \pi^+\pi^0\gamma$.

$K_L \rightarrow \pi^+\pi^-\gamma$. Bremsstrahlung (E_B) is suppressed by CP violation but enhanced by the $1/E_\gamma$ behaviour. KTeV has also measured the magnetic transition $M1$ with a non-trivial form factor [48]:

$$M1 = \tilde{g}_{M1} \left[1 + \frac{a}{1 - M_K^2/M_\rho^2 + 2M_K E_\gamma^*/M_\rho^2} \right] \quad (17)$$

determining $a = (-1.243 \pm 0.057)$ and the branching given in the table, which fixes also \tilde{g}_{M1} . Interestingly they find that this parametrization is substantially better than a linear fit showing that VMD is at work. In terms of the basic $\mathcal{O}(p^4)$ weak lagrangian in (1) $M1$ is written as

$$M1 = N_{29} + N_{31} + \text{h.o.} \quad (18)$$

There are two ways of implementing VMD in (1) with different results for $M1$ [13, 14]. KTeV data in (17) have shown that there are large VMD contributions to (18) and so (analogously to the strong sector), data prefer that VMD be realized at $\mathcal{O}(p^4)$ [14, 49], and not $\mathcal{O}(p^6)$ [50].

$K_{L,S} \rightarrow \pi^+\pi^-e^+e^-$ KTeV and NA48 [51, 4] have recently measured the asymmetry in the angle between the e^+e^- and the $\pi^+\pi^-$ planes in the decay $K_L \rightarrow \pi^+\pi^-e^+e^-$: this measures the CP violating interference of the bremsstrahlung (E_B) with the $M1$ transition of $K_L \rightarrow \pi^+\pi^-\gamma$ decays, enhanced by the CP-suppressed denominator $\Gamma(K_L \rightarrow \pi^+\pi^-e^+e^-)$ ($\sim E_B^2$). This quantity is thus very well predicted in terms of known long-distance observables, but it is not an efficient CKM test [52]. Recently NA48 has measured the CP-even bremsstrahlung dominated decay $K_S \rightarrow \pi^+\pi^-e^+e^-$ [53]. The asymmetry in the angle between the e^+e^- and the $\pi^+\pi^-$ planes for $K_S \rightarrow \pi^+\pi^-e^+e^-$ is small; however, it might be interesting for NA48 and KLOE to test different observables [54].

$K^+ \rightarrow \pi^+\pi^0\gamma$. Due to the $\Delta I = 3/2$ suppression of the bremsstrahlung, interference between E_B and $E1$ and magnetic transitions can be measured. New data from BNL E787 [55] show vanishing interference, thus putting a non-trivial bound on model predictions for the counterterm coefficient in (1) contributing to $E1$ [13, 14]. Consequently the direct emission branching $(B(K^+ \rightarrow \pi^+\pi^0\gamma)_{\text{exp}}^{DE})$, in the table, must be interpreted as a pure magnetic transition and related to the analogous one in $K_L \rightarrow \pi^+\pi^-\gamma$ [49].

Direct CP violation Direct CP violation can be established in the width charge asymmetry in $K^\pm \rightarrow \pi^\pm\pi^0\gamma$, $\delta\Gamma/2\Gamma$, and in the interference E_B with $E1$ in $K_L \rightarrow \pi^+\pi^-\gamma$ ($E1$ with M_1 in $K_L \rightarrow \pi^+\pi^-e^+e^-$); both observables are kinematically difficult since one is looking in the Dalitz plot at large photon energy [11]. SM charge asymmetries were looked in [56] expecting $\delta\Gamma/2\Gamma \leq 10^{-5}$. Supersymmetry may enhance this asymmetry by a factor of 10 [57].

6 $K_L \rightarrow l^+l^-$

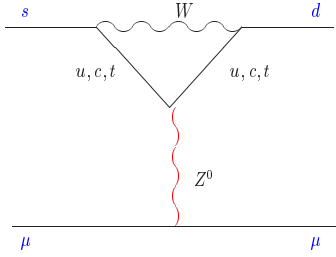


Figure 7: A_{SD}

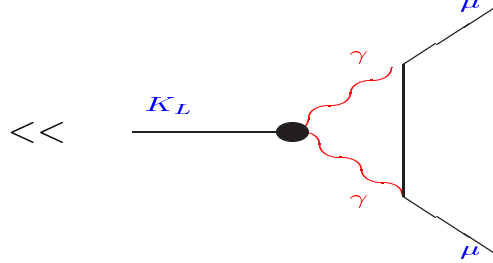


Figure 8: A_{LD}

$K_L \rightarrow \mu^+\mu^-$ is an interesting channel to determine V_{td} and to probe new physics. In the Standard Model the short-distance contribution, A_{SD} , is generated by diagrams like the one in Fig. 7. The known experimental rate $\Gamma(K_L \rightarrow \gamma\gamma)$ [58] allows us to determine the two-photon absorptive contribution, $|\Im A|^2$, in Fig. 8 [59]. This almost saturates the experimental $K_L \rightarrow \mu^+\mu^-$ rate from E871 [58, 60]:

$$\text{Br}(K_L \rightarrow \mu\bar{\mu}) = |\Re A|^2 + |\Im A|^2 \stackrel{\text{E871}}{=} (7.18 \pm 0.17) \times 10^{-9} \quad (19)$$

$$\swarrow \quad \begin{array}{c} B(K_L \rightarrow \gamma\gamma) \\ \downarrow \end{array} \quad \Re(A_{\text{SD}} + A_{\text{LD}}), \quad |\Im A|^2 = (7.1 \pm 0.2) \times 10^{-9} \quad (20)$$

Thus the sum of the real parts, long and short distance, $\Re(A) = \Re(A_{\text{SD}} + A_{\text{LD}})$, is bound to be very small: $|\Re(A_{\text{exp}})|^2 < 4.0 \times 10^{-10}$ at 90% C.L. The known

V_{td} -dependence of the SM short-distance amplitude A_{SD} [2, 61] allows us to obtain the bound on $\bar{\rho} = \rho(1 - \lambda^2/2)$ [62]:

$$\bar{\rho} > 1.2 - \max \left[\frac{|\Re A_{\text{exp}}| + |\Re A_{LD}|}{3 \times 10^{-5}} \left(\frac{\bar{m}_t(m_t)}{170 \text{ GeV}} \right)^{-1.55} \left(\frac{|V_{cb}|}{0.040} \right)^{-2} \right] \quad (21)$$

To do better and constrain new physics it is necessary to have a reliable control on the model-dependent long-distance dispersive amplitude A_{LD} in Fig. 8 [62]-[64]. In practice one has to understand the proper $K_L \rightarrow \gamma^* \gamma^*$ form factor, $f(q_1^2, q_2^2)$, for Fig. 8. In Ref. [64], in analogy to the real decay $K_L \rightarrow \gamma \gamma$, this has first been written as the sum of the poles π^0, η and η' ; then weak couplings are determined by a large- N_c argument and $U(3) \otimes U(3)$ symmetry, while the experimental knowledge of the electromagnetic decays of pseudoscalars, $P, P \rightarrow e^+ e^-$ constrains the relevant local contribution. Somehow the problem of the form factor has been thus bypassed. However we think the form factor is very sensitive to symmetry breaking and thus caution must be used before completely accepting this result [15]. We instead have proposed [62] a low energy parameterization of the $K_L \rightarrow \gamma^* \gamma^*$ form factor that includes the poles of the lowest vector meson resonances ($m_V \sim m_\rho$):

$$f(q_1^2, q_2^2) = 1 + \alpha \left(\frac{q_1^2}{q_1^2 - m_V^2} + \frac{q_2^2}{q_2^2 - m_V^2} \right) + \beta \frac{q_1^2 q_2^2}{(q_1^2 - m_V^2)(q_2^2 - m_V^2)}. \quad (22)$$

The ansatz is that, since we are able to describe the relevant resonances fully, this is the proper form factor to high energy up to the charm scale and in fact by comparing it with the short-distance result in [61] we constrain the parameters α and β : the form factor in Eq. (22) goes as $1 + 2\alpha + \beta$ for $q_i^2 \gg m_V^2$ and thus the logarithmically divergent A_{LD} in Fig. 8 can be phenomenologically compared with the known perturbative QCD calculation [61] leading to $|1 + 2\alpha + \beta| \ln(\Lambda/m_V) < 0.4$ (Λ is the ultraviolet cutoff) and limiting β for a fixed value of α . There are two important questions that we will address now to establish A_{SD} accurately and safely: i) the experimental or theoretical determination of the parameters α and β (expected to be $\mathcal{O}(1)$ by chiral power counting) and ii) making sure that the form factor in Eq. (22) is correct. We can expand Eq. (22) for $q_2^2 = 0$ and $q_1^2 \ll m_V^2$, obtaining

$$f(q_1^2 \ll m_V^2, q_2^2 = 0) = 1 - 0.42\alpha \frac{q_1^2}{m_K^2} - 0.17\alpha \frac{q_1^4}{m_K^4} + \text{h.o.} \quad (23)$$

and describe simultaneously $K_L \rightarrow e^+ e^- \gamma$ and $K_L \rightarrow \mu^+ \mu^- \gamma$ decays. However, data are not yet sufficient to clearly show if Eq. (23) is a better description of these decays than, for example, a linear fit. Historically data have been analysed

using simply the BMS form factor [65], this is still VMD motivated and, in the low energy region it is similar to the form factor (23), but for $q^2 \gg m_V^2$ cannot match QCD and thus must be regarded as a low energy phenomenological model. The low energy parametrization of the BMS model is:

$$f_{BMS}(q_1^2 \ll m_V^2) = 1 + (0.42 - 1.3\alpha_{K^*})\frac{q_1^2}{m_K^2} + (0.17 - 0.91\alpha_{K^*})\frac{q_1^4}{m_K^4} + \text{h.o.} \quad (24)$$

In fact $K_L \rightarrow e^+e^-\gamma$ [66] has been analysed using only (24) and finding $\alpha_{K^*} = (-0.36 \pm 0.1)$. I have checked that (23) with $\alpha = -1.5$ fits even better the $K_L \rightarrow e^+e^-\gamma$ spectrum. KTeV has recently measured the $K_L \rightarrow \mu^+\mu^-\gamma$ spectrum and rate with (23) and (24), finding respectively $\alpha = -1.54 \pm 0.10$ and $\alpha_{K^*} = -0.160_{-0.028}^{+0.026}$ [67]: for these values even the quadratic slopes in (23) and (24) agree. However it seems that the BMS model does not fit simultaneously $K_L \rightarrow e^+e^-\gamma$ and $K_L \rightarrow \mu^+\mu^-\gamma$ spectra but this could be also caused by some experimental problem. We look forward to a clear determination of the linear and quadratic slopes in both lepton channels, so to clearly establish that the form factors in (23) and (24) are better than the linear slope. For the values in Ref.[67] the difference is marginal. We stress that the advantage of our model is the good behaviour for large q^2 . Another important test is the measurement of the quadratic slope β in (22) from $K_L \rightarrow e^+e^-\mu^+\mu^-$ [70] or $K_L \rightarrow e^+e^-e^+e^-$ [69, 68]. Of course this is a difficult measurement; however, encouraging results have been obtained lately in $e^+e^-\mu^+\mu^-$ (43 events) by KTeV [70] and $e^+e^-e^+e^-$ (441 events) by KTeV [69] and by NA48 [68], where the branchings have been obtained and although β is not determined yet, the measurement of the form factor is not so far away since the linear terms have already been studied. [69]

Now, if we take: β from the matching conditions and the latest experimental determinations of $\Gamma(K_L \rightarrow \mu^+\mu^-)$, $\Gamma(K_L \rightarrow \gamma\gamma)$ and α , we obtain $|\Re\mathcal{A}_{LD}| < 2.07 \times 10^{-5}$ and $\rho > -0.2$ at 90% *C.L.* [67, 71]. This bound could be improved and made more solid if the form factor in Eq. (22) were firmly established and the parameters were measured with good precision. An encouraging result is also that the experimental value for α follows the theoretical prediction in Ref.[32] (see also Eq. (9)):

$$\mathcal{L}_W^{FMV}(\phi, V^\mu) \implies \alpha = 1.2 \quad (25)$$

based on short distance, showing that the low energy description in Eq. (22) is able to capture also short-distance physics. Also lattice [72] might help to establish the correct form factor. Recently $K_L \rightarrow e^+e^-$ has been measured at BNL by E871 as [73]: $B(K_L \rightarrow e^+e^-) = (8.7_{-4.1}^{+5.7}) \times 10^{-12}$; however, the theoretical prediction [63, 64] for this branching is not sensitive to the slopes of the form factor but only to $f(0, 0)$.

7 Conclusions

I think that we have heard at this Conference and I have summarized here some relevant progress: the improved measurements of $K_S \rightarrow \gamma\gamma$, $K_L \rightarrow \pi^0\gamma\gamma$, $K_L \rightarrow \pi^+\pi^-\gamma$ and $K_L \rightarrow l^+l^-\gamma(\gamma^*)$ decays. These are useful pieces of information, which will serve to improve our ability in testing the SM and understand QCD. Soon we will have interesting data from KLOE, NA48b and E949 [4, 6, 8] so that other channels such as $K_S \rightarrow \pi^0e^+e^-$, $K^+ \rightarrow \pi^+\gamma\gamma$ and interesting CP-violating asymmetries, e.g. the charge asymmetry in $K^+ \rightarrow \pi^+\pi^0\gamma$ and $K^+ \rightarrow \pi^+\mu^+\mu^-$ [7], will be measured. We have seen that our ability to test the SM in $K_L \rightarrow \pi^0e^+e^-$ and $K_L \rightarrow \mu^+\mu^-$ depends crucially on how good we match short distance: here theoretical progress has been made [15] and more is needed. Other interesting prospects are the muon polarization in $K_L \rightarrow \pi^0\mu^+\mu^-$ [24] and time interferences in $K_{L,S} \rightarrow \pi^0e^+e^-$ [74] to definitely suppress Greenlee background.

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